Lecture 8

Set Sprinkler Lateral Design

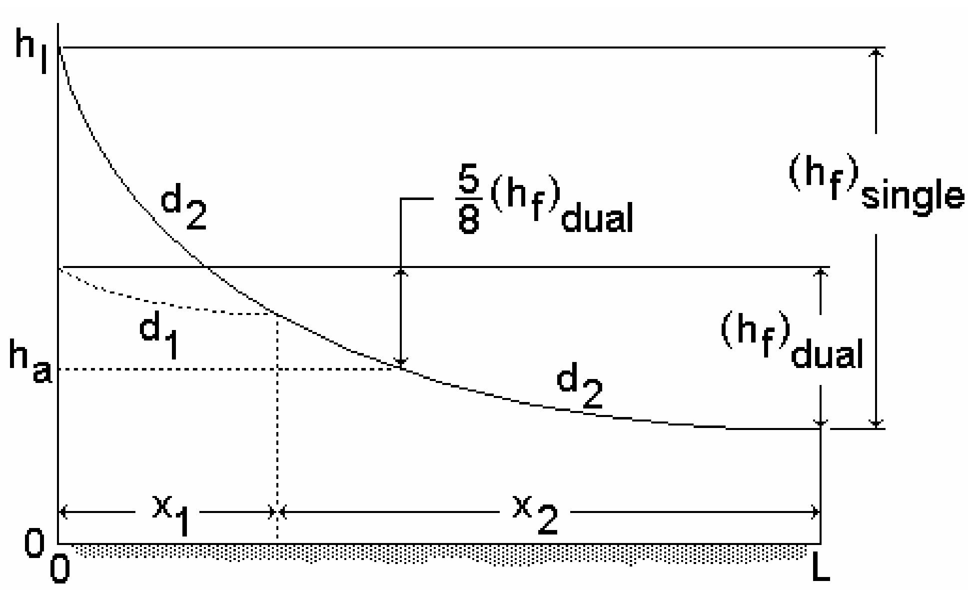
I. Dual Pipe Size Laterals

- Sometimes it is useful to design a lateral pipe with two different diameters to accomplish either of the following:

  1. a reduction in h_f
  2. an increase in h_f

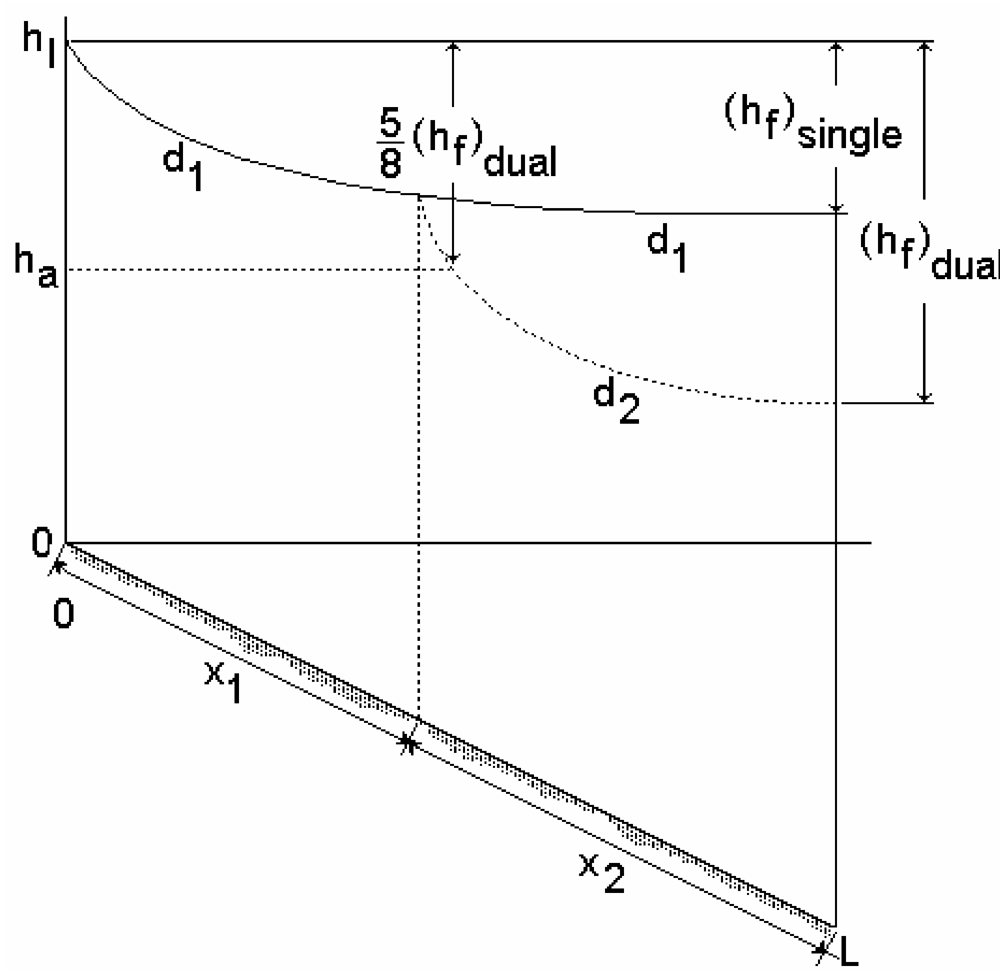
- In either case, the basic objective is to reduce pressure variations along the lateral pipe by arranging the friction loss curve so that it *more closely parallels the ground slope*
- It is not normally desirable to have more than one pipe size in portable laterals (hand-move, wheel lines), because it usually makes set changes more troublesome
- For fixed systems with buried laterals, it may be all right to have more than two pipe diameters along the laterals
- For dual pipe size laterals, approximately 5/8 of the pressure loss due to friction occurs between the lateral inlet and the location of average pressure

- **Case 1**: a lateral on level ground where one pipe size is too small, but the next larger size is too big...



- d_1 is the larger diameter, and d_2 is the smaller diameter
- note that $(h_f)_{single}$ is much larger than $(h_f)_{dual}$

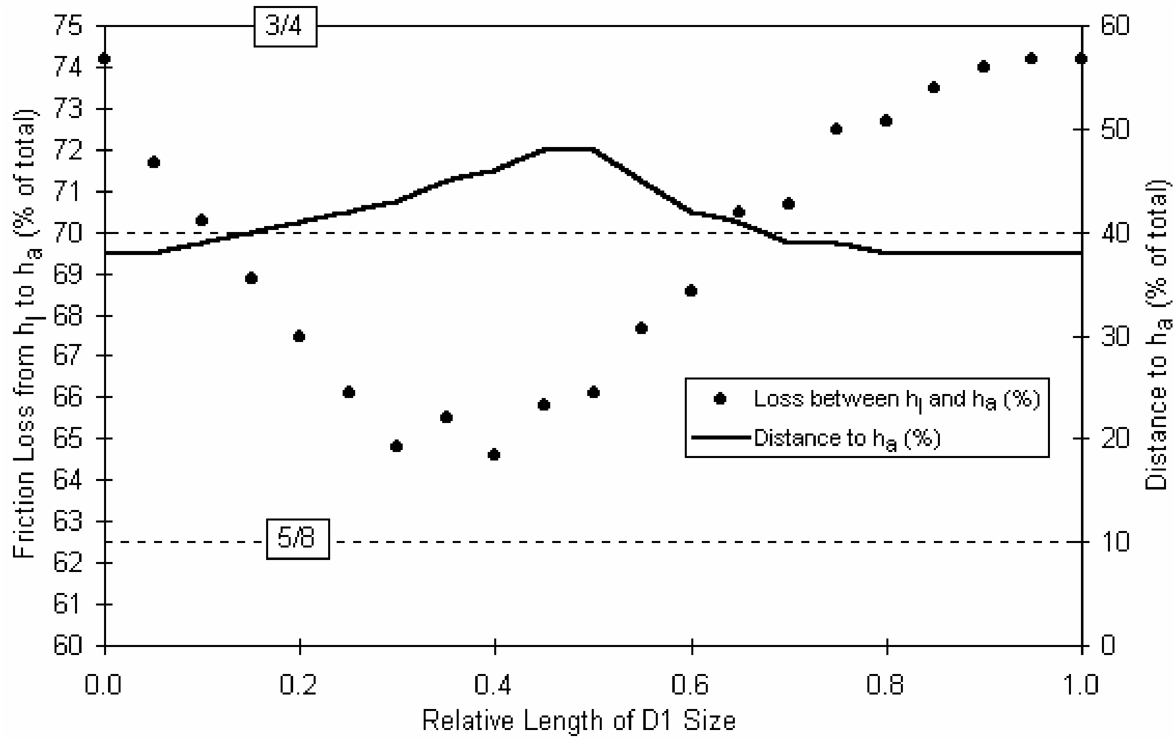
- **Case 2**: a lateral running downhill where one pipe size is too big, but the next smaller size is too small...



- The composite friction loss curve for d_1 and d_2 more closely parallels the ground slope than the curve with only d_1, which means that the pressure variation along the lateral is less with the dual pipe size design

II. Location of Average Pressure in Dual Size Laterals

- Do you believe that $5/8(h_f)_{dual}$ occurs between h_l and h_a?
- Consider the analysis shown graphically below (5/8 = 0.625)
- The plot is for a dual pipe size lateral with D_1 = 15 cm, D_2 = 12 cm, 100 equally-spaced outlets, 900 m total lateral length, Hazen-Williams C factor of 130, uniform sprinkler flow rate of 0.4 lps, and zero ground slope



- Notice where the ¾ value is on the left-hand ordinate
- Notice that the head loss from h_l to h_a is approximately 74% when the pipe size is all D_1 (1.0 on the abscissa) and when the pipe size is all D_2 (0.0 on the abscissa)
- The h_l values (inlet pressure head) would be different for each point on the curve if it were desired to maintain the same h_a for different lateral designs
- Notice that the distance from the lateral inlet to the location of average pressure head is roughly 40% of the total lateral length, but varies somewhat depending on the ratio of lengths of D_1 to D_2 (in this example)

- These calculations can be set up on a spreadsheet to analyze any particular combination of pipe sizes and other hydraulic conditions. Below is an example:

Section	Flow (lps)	Distance (m)	Diameter (cm)	h_f (m)	Sum (h_f) (m)	d(h_e) (m)	head (m)	diff from ha (%)	$h_f/(h_f)_{total}$
1	40.00	9.00	15.00	0.31	0.31	0	49.69	12.08	0.016
2	39.60	18.00	15.00	0.31	0.62	0	49.38	11.77	0.032
3	39.20	27.00	15.00	0.30	0.92	0	49.08	11.47	0.048
4	38.80	36.00	15.00	0.29	1.21	0	48.79	11.18	0.064
5	38.40	45.00	15.00	0.29	1.50	0	48.50	10.89	0.079
6	38.00	54.00	15.00	0.28	1.79	0	48.21	10.61	0.094

III. Determining X_1 and X_2 in Dual Pipe Size Laterals

- The friction loss is:

$$h_f = \left(\frac{J_1F_1L}{100} - \frac{J_2F_2x_2}{100}\right) + \left(\frac{J_3F_2x_2}{100}\right) \tag{132}$$

where,

h_f = total lateral friction head loss for dual pipe sizes

J_1 = friction loss gradient for D_1 and Q_{inlet}
J_2 = friction loss gradient for D_1 and Q_{inlet} - $(q_a)(x_1)/S_e$
J_3 = friction loss gradient for D_2 and Q_{inlet} - $(q_a)(x_1)/S_e$

F_1 = multiple outlet reduction coefficient for L/S_e outlets
F_2 = multiple outlet reduction coefficient for x_2/S_e outlets

x_1 = length of D_1 pipe (larger size)
x_2 = length of D_2 pipe (smaller size)

$x_1 + x_2 = L$

- As in previous examples, we assume constant q_a
- As for single pipe size laterals, we will fix h_f by

$$\Delta h = h_f + \Delta h_e = 20\% h_a \tag{133}$$

and,

$$h_f = 20\% h_a - \Delta h_e \tag{134}$$

- Find d_1 and d_2 in tables (or by calculation) using Q_{inlet} and...

$$(J)_{d1} \le J_a \le (J)_{d2} \tag{135}$$

for,

$$J_a = 100\left(\frac{20\% h_a - \Delta h_e}{FL}\right) \tag{136}$$

- Now there are two adjacent pipe sizes: d_1 and d_2
- Solve for x_1 and x_2 by trial-and-error, or write a computer program, and make $h_f = 0.20h_a - \Delta h_e$ (you already have an equation for h_f above)

IV. Setting up a Computer Program to Determine X_1 and X_2

- If the Hazen-Williams equation is used, the two F values will be:

$$F_1 \approx 0.351 + \frac{1}{2N_1}\left(1 + \frac{4}{13N_1}\right) \quad (137)$$

$$F_2 \approx 0.351 + \frac{1}{2N_2}\left(1 + \frac{4}{13N_2}\right) \quad (138)$$

where

$$N_1 = \frac{L}{S_e} \quad (139)$$

$$N_2 = \frac{L - x_1}{S_e} \quad (140)$$

- The three friction loss gradients are:

$$J_1 = K\left(\frac{Q_1}{C}\right)^{1.852} D_1^{-4.87} \quad (141)$$

$$J_2 = K\left(\frac{Q_2}{C}\right)^{1.852} D_1^{-4.87} \quad (142)$$

$$J_3 = K\left(\frac{Q_2}{C}\right)^{1.852} D_2^{-4.87} \quad (143)$$

where

$$Q_1 = \left(\frac{L}{S_e}\right) q_a \quad (144)$$

$$Q_2 = \left(\frac{L - x_1}{S_e}\right) q_a \quad (145)$$

- The coefficient K in Eqs. 141-143 is 1,050 for gpm & inches; $16.42(10)^6$ for lps and cm; or $1.217(10)^{12}$ for lps and mm

- Combine the above equations and set it equal to zero:

$$f(x_1) = \alpha_1 \left[\alpha_2 - \alpha_3 (L - x_1)^{2.852} F_2 \right] - 0.2h_a + \Delta h_e = 0 \qquad (146)$$

where

$$\alpha_1 = \frac{K}{100\,C^{1.852}} \qquad (147)$$

$$\alpha_2 = \left(\frac{q_a L}{S_e} \right)^{1.852} D_1^{-4.87} F_1 L \qquad (148)$$

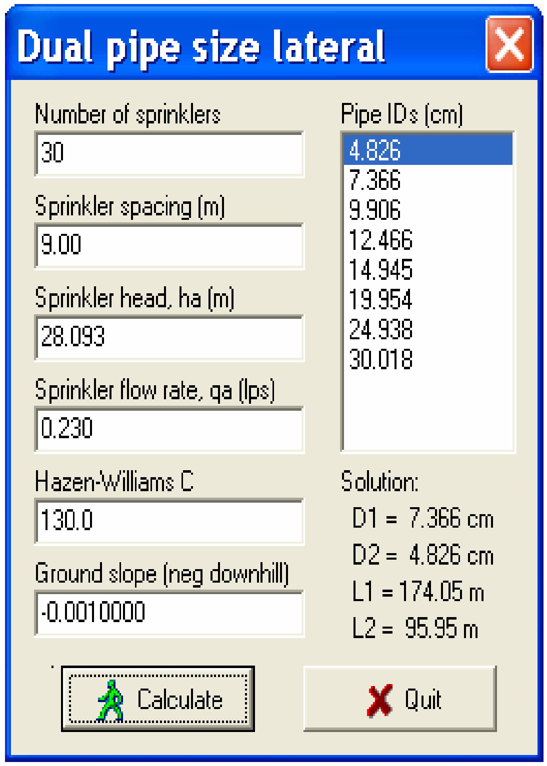
$$\alpha_3 = \left(D_1^{-4.87} - D_2^{-4.87} \right) \left(\frac{q_a}{S_e} \right)^{1.852} \qquad (149)$$

- The three alpha values are constants
- Eq. 146 can be solved for the unknown, x_1, by the Newton-Raphson method
- To accomplish this, we need the derivative of Eq. 146 with respect to x_1

$$\frac{df(x_1)}{dx_1} = $$

$$= \alpha_1 \alpha_3 \left[2.852 F_2 (L - x_1)^{1.852} - \frac{S_e (L - x_1)^{0.852}}{2} \left(1 + \frac{8 S_e}{13(L - x_1)} \right) \right] \qquad (150)$$

- Note that the solution may fail if the sizes D_1 & D_2 are inappropriate
- To make things more interesting, give the computer program a list of inside pipe diameters so that it can find the most appropriate available values of D_1 & D_2
- Note that the Darcy-Weisbach equation could be used instead of Hazen-Williams
- In Eq. 146 you could adjust the value of the 0.2 coefficient on h_a to determine its sensitivity to the pipe diameters and lengths
- The following screenshot is of a small computer program for calculating diameters and lengths of dual pipe size sprinkler laterals



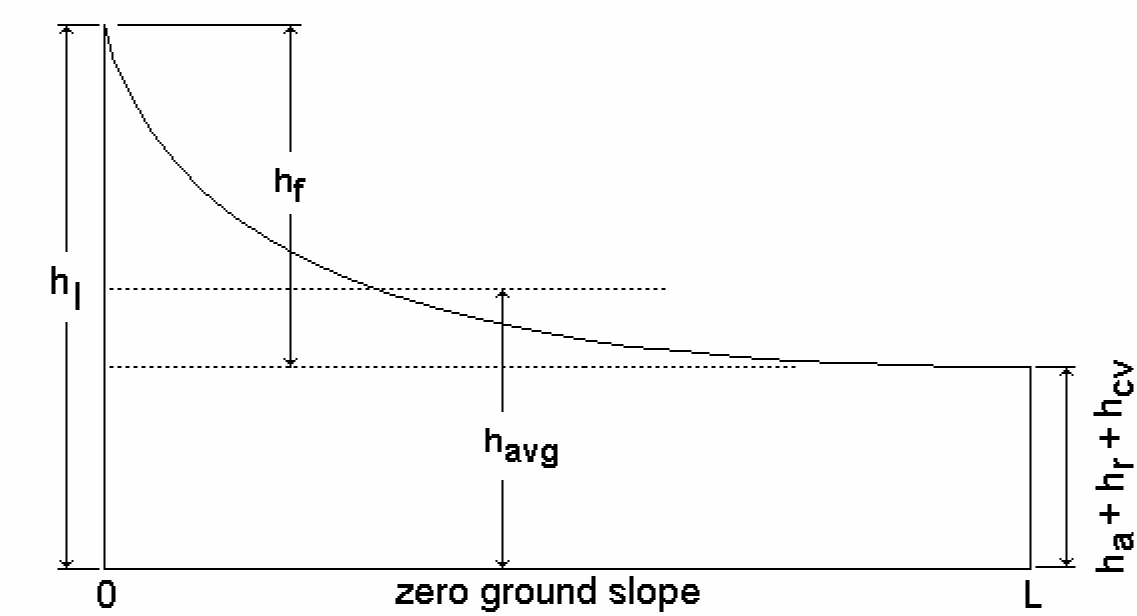
V. Inlet Pressure for Dual Pipe Size Laterals

$$h_l = h_a + \frac{5}{8}h_f + \frac{1}{2}\Delta h_e + h_r \tag{151}$$

- This is the same as the lateral inlet pressure head equation for single pipe size, except that the coefficient on h_f is 5/8 instead of 3/4
- Remember that for a downhill slope, the respective pressure changes due to friction loss and due to elevation change are <u>opposing</u>

VI. Laterals with Flow Control Devices

- Pressure regulating valves can be located at the base of each sprinkler: These have approximately 2 to 5 psi (14 to 34 kPa) head loss
- Also, flow control nozzles (FCNs) can be installed in the sprinkler heads
- FCNs typically have negligible head loss
- For a lateral on level ground, the minimum pressure is at the end:



- The lateral inlet pressure head, h_l, is determined such that the minimum pressure in the lateral is enough to provide h_a at each sprinkler...

$$h_l = h_a + h_f + \Delta h_e + h_r + h_{cv} \tag{152}$$

where h_{cv} is the pressure head loss through the flow control device

- For a lateral with flow control devices, the average pressure is not equal to the nominal sprinkler pressure

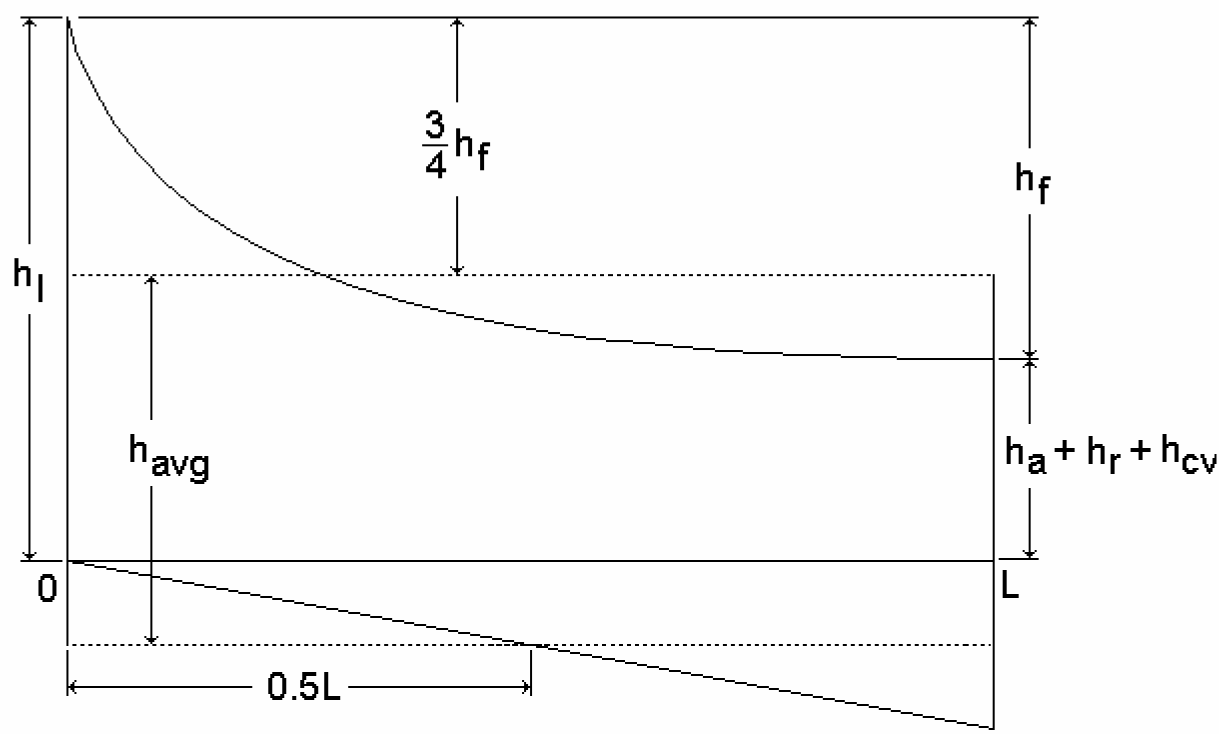
$$h_{avg} \neq h_a \tag{153}$$

- If the pressure in the lateral is enough everywhere, then

$$h_a = \left(\frac{q_a}{K_d} \right)^2 \tag{154}$$

where h_a is the pressure head at the sprinklers

- Below is a sketch of the hydraulics for a downhill lateral with flow control devices



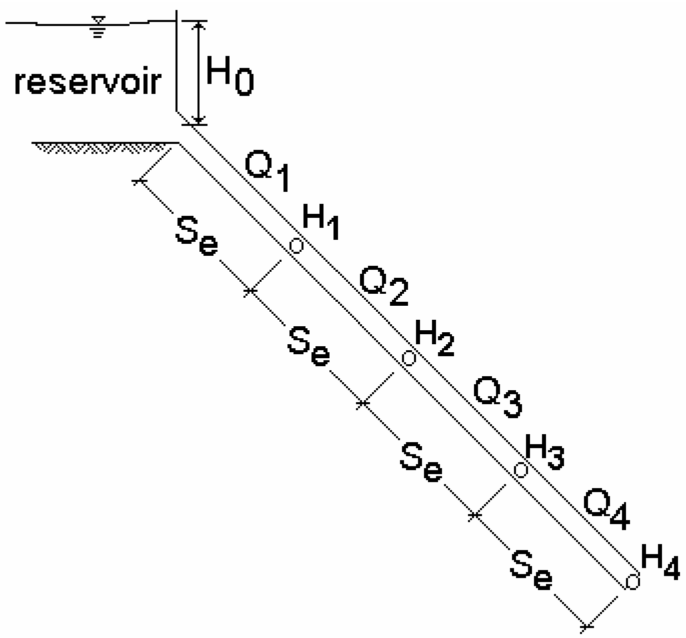
VII. Anti-Drain Valves

- Valves are available for preventing flow through sprinklers until a certain minimum pressure is reached
- These valves are installed at the base of each sprinkler and are useful where sprinkler irrigation is used to germinate seeds on medium or high value crops
- The valves help prevent seed bed damage due to low pressure streams of water during startup and shutdown
- But, for periodic-move, the lines still must be drained before moving

Gravity-Fed Lateral Hydraulic Analysis

I. Description of the Problem

- A gravity-fed sprinkler lateral with evenly spaced outlets (sprinklers), beginning at a distance S_e from the inlet:



- The question is, for known inlet head, H_0, pipe diameter, D, sprinkler spacing, S_e, ground slope, S_o, sprinkler discharge coefficient, K_d, riser height, h_r, and pipe material (C factor), what is the flow rate through each sprinkler?
- Knowing the answer will lead to predictions of application uniformity
- In this case, we won't assume a constant q_a at each sprinkler

II. Friction Loss in the Lateral

Hazen-Williams equation:

$$h_f = \frac{JL}{100} \tag{155}$$

$$J = 16.42(10)^6 \left(\frac{Q}{C}\right)^{1.852} D^{-4.87} \tag{156}$$

for Q in lps; D in cm; J in m/100 m; L in m; and h_f in m.

- Between two sprinklers,

$$h_f = \frac{JS_e}{100} = 16.42(10)^4 S_e \left(\frac{Q}{C}\right)^{1.852} D^{-4.87} \quad (157)$$

or,

$$h_f = h_w Q^{1.852} \quad (158)$$

where Q is the flow rate in the lateral pipe between two sprinklers, and

$$h_w = 16.42(10)^4 S_e C^{-1.852} D^{-4.87} \quad (159)$$

III. Sprinkler Discharge

typically,

$$q = K_d \sqrt{h} \quad (160)$$

where q is the sprinkler flow rate in lps; h is the pressure head at the sprinkler in m; and K_d is an empirical coefficient: $K_d = K_o A$, where A is the cross sectional area of the inside of the pipe

IV. Develop the System of Equations

- Suppose there are only four sprinklers, evenly spaced (see the above figure)
- Suppose that we know H_0, K_d, C, D, h_r, S_o, and S_e

$$q_1 = K_d\sqrt{H_1 - h_r} \quad \rightarrow \quad H_1 = h_r + \left(\frac{q_1}{K_d}\right)^2 = h_r + \frac{(Q_1 - Q_2)^2}{K_d^2} \quad (161)$$

$$q_2 = K_d\sqrt{H_2 - h_r} \quad \rightarrow \quad H_2 = h_r + \left(\frac{q_2}{K_d}\right)^2 = h_r + \frac{(Q_2 - Q_3)^2}{K_d^2} \quad (162)$$

$$q_3 = K_d\sqrt{H_3 - h_r} \quad \rightarrow \quad H_3 = h_r + \left(\frac{q_3}{K_d}\right)^2 = h_r + \frac{(Q_3 - Q_4)^2}{K_d^2} \quad (163)$$

$$q_4 = K_d\sqrt{H_4 - h_r} \quad \rightarrow \quad H_4 = h_r + \left(\frac{q_4}{K_d}\right)^2 = h_r + \frac{{Q_4}^2}{K_d^2} \quad (164)$$

- Pressure heads can also be defined independently in terms of friction loss along the lateral pipe

$$H_1 = H_0 - h_w Q_1^{1.852} - \Delta h_e \tag{165}$$

$$H_2 = H_1 - h_w Q_2^{1.852} - \Delta h_e \tag{166}$$

$$H_3 = H_2 - h_w Q_3^{1.852} - \Delta h_e \tag{167}$$

$$H_4 = H_3 - h_w Q_4^{1.852} - \Delta h_e \tag{168}$$

where,

$$\Delta h_e = \frac{S_e}{\sqrt{S_o^{-2} + 1}} \tag{169}$$

and S_o is the ground slope (m/m)

- The above presumes a uniform, constant ground slope
- Note that in the above equation, Δh_e is always positive. So it is necessary to multiply the result by –1 (change the sign) whenever $S_o < 0$.
- Note also that $S_o < 0$ means the lateral runs in the downhill direction
- Combining respective H equations:

$$\frac{(Q_1 - Q_2)^2}{K_d^2} = H_0 - h_w Q_1^{1.852} - \Delta h_e - h_r \tag{170}$$

$$\frac{(Q_2 - Q_3)^2}{K_d^2} = H_1 - h_w Q_2^{1.852} - \Delta h_e - h_r \tag{171}$$

$$\frac{(Q_3 - Q_4)^2}{K_d^2} = H_2 - h_w Q_3^{1.852} - \Delta h_e - h_r \tag{172}$$

$$\frac{Q_4^2}{K_d^2} = H_3 - h_w Q_4^{1.852} - \Delta h_e - h_r \tag{173}$$

- Setting the equations equal to zero:

$$f_1 = H_0 - \frac{(Q_1 - Q_2)^2}{K_d^2} - h_w Q_1^{1.852} - \Delta h_e - h_r = 0 \qquad (174)$$

$$f_2 = \frac{(Q_1 - Q_2)^2}{K_d^2} - \frac{(Q_2 - Q_3)^2}{K_d^2} - h_w Q_2^{1.852} - \Delta h_e - h_r = 0 \qquad (175)$$

$$f_3 = \frac{(Q_2 - Q_3)^2}{K_d^2} - \frac{(Q_3 - Q_4)^2}{K_d^2} - h_w Q_3^{1.852} - \Delta h_e - h_r = 0 \qquad (176)$$

$$f_4 = \frac{(Q_3 - Q_4)^2}{K_d^2} - \frac{{Q_4}^2}{K_d^2} - h_w Q_4^{1.852} - \Delta h_e - h_r = 0 \qquad (177)$$

The system of equations can be put into matrix form as follows:

$$\begin{bmatrix} \frac{\partial f_1}{\partial Q_1} & \frac{\partial f_1}{\partial Q_2} & & \\ \frac{\partial f_2}{\partial Q_1} & \frac{\partial f_2}{\partial Q_2} & \frac{\partial f_2}{\partial Q_3} & \\ & \frac{\partial f_3}{\partial Q_2} & \frac{\partial f_3}{\partial Q_3} & \frac{\partial f_3}{\partial Q_4} \\ & & \frac{\partial f_4}{\partial Q_3} & \frac{\partial f_4}{\partial Q_4} \end{bmatrix} \begin{bmatrix} \delta Q_1 \\ \delta Q_2 \\ \delta Q_3 \\ \delta Q_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \qquad (178)$$

where the two values in the first row of the square matrix are:

$$\frac{\partial f_1}{\partial Q_1} = \frac{-2(Q_1 - Q_2)}{K_d^2} - 1.852 h_w Q_1^{0.852} \qquad (179)$$

$$\frac{\partial f_1}{\partial Q_2} = \frac{2(Q_1 - Q_2)}{K_d^2} \qquad (180)$$

The two values in the last row of the square matrix, for n sprinklers, are:

$$\frac{\partial f_n}{\partial Q_{n-1}} = \frac{2\left(Q_{n-1} - Q_n\right)}{K_d^2} \tag{181}$$

$$\frac{\partial f_n}{\partial Q_n} = \frac{-2Q_{n-1}}{K_d^2} - 1.852 h_w Q_n^{0.852} \tag{182}$$

and the three values in each intermediate row of the matrix are:

$$\frac{\partial f_i}{\partial Q_{i-1}} = \frac{2\left(Q_{i-1} - Q_i\right)}{K_d^2} \tag{183}$$

$$\frac{\partial f_i}{\partial Q_i} = \frac{2\left(Q_{i+1} - Q_{i-1}\right)}{K_d^2} - 1.852 h_w Q_i^{0.852} \tag{184}$$

$$\frac{\partial f_i}{\partial Q_{i+1}} = \frac{2\left(Q_i - Q_{i+1}\right)}{K_d^2} \tag{185}$$

where i is the row number

- This is a system of nonlinear algebraic equations
- The square matrix is a *Jacobian matrix*; all blank values are zero
- Solve for Q_1, Q_2, Q_3, and Q_4 (or up to Q_n, in general) using the Newton-Raphson method, Gauss elimination, and backward substitution (or other solution method for a linear set of equations)
- Knowing the flow rates, you can go back and directly calculate the pressure heads one by one

- The problem could be further generalized by allowing for different pipe sizes in the lateral, by including minor losses, by allowing variable elevation changes between sprinkler positions, etc.
- However, it is still a problem of solving for x unknowns and x equations
- For pumped systems (not gravity, as above), we could include a mathematical representation of the pump characteristic curve to determine the lateral hydraulic performance; that is, don't assume a constant H_0, but replace it by some function

V. Brute-Force Approach

- There is a computer program that will do the above calculations for a gravity-fed lateral with multiple sprinklers
- But, if you want to write your own program in a simpler way, you can do the calculations by "brute-force" as follows:

1. Guess the pressure at the end of the lateral
2. Calculate q for the last sprinkler
3. Calculate h_f over the distance S_e to the next sprinkler upstream
4. Calculate Δh_e over the same S_e
5. Get the pressure at that next sprinkler and calculate the sprinkler flow rate
6. Keep moving upstream to the lateral inlet
7. If the head is more than the available head, reduce the end pressure and start over, else increase the pressure and start over

- Below is a screenshot of a computer program that will do the above calculations for a gravity-fed lateral with multiple sprinklers