**Lecture 6**

**Economic Pipe Selection Method**

I. Introduction

- The *economic pipe selection method* (Chapter 8 of the textbook) is used to balance fixed (initial) costs for pipe with annual energy costs for pumping.
- With larger pipe sizes the average flow velocity for a given discharge decreases, causing a corresponding decrease in friction loss.
- This reduces the head on the pump, and energy can be saved.
- However, larger pipes cost more to purchase.

<table>
<thead>
<tr>
<th>Pipe Size (diameter)</th>
<th>Cost</th>
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<tbody>
<tr>
<td></td>
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</table>

To balance these costs and find the minimum cost we will annualize the fixed costs, compare with annual energy (pumping) costs.

We can also graph the results so that pipe diameters can be selected according to their maximum flow rate.

We will take into account interest rates and inflation rates to make the comparison.

This is basically an “engineering economics” problem, specially adapted to the selection of pipe sizes.

This method involves the following principal steps:

1. Determine the equivalent annual cost for purchasing each available pipe size
2. Determine the annual energy cost of pumping
3. Balance the annual costs for adjacent pipe sizes
4. Construct a graph of system flow rate versus section flow rate on a log-log scale for adjacent pipe sizes

- We will use the method to calculate “cut-off” points between adjacent pipe sizes so that we know which size is more economical for a particular flow rate
- We will use HP and kW units for power, where about ¾ of a kW equals a HP
- Recall that a Watt (W) is defined as a joule/second, or a N-m per second
- Multiply W by elapsed time to obtain Newton-meters (“work”, or “energy”)

II. Economic Pipe Selection Method Calculations

1. Select a period of time over which comparisons will be made between fixed and annual costs. This will be called the $useful\ life$ of the system, $n$, in years.

   - The “useful life” is a subjective value, subject to opinion and financial amortization conditions
   - This value could alternatively be specified in months, or other time period, but the following calculations would have to be consistent with the choice

2. For several different pipe sizes, calculate the $uniform\ annual\ cost$ of pipe per unit length of pipe.

   - A unit length of 100 (m or ft) is convenient because $J$ is in $m/100\ m$ or $ft/100\ ft$, and you want a fair comparison (the actual pipe lengths from the supplier are irrelevant for these calculations)
   - You must use consistent units ($$/100\ ft$$ or $$$/100\ m$$) throughout the calculations, otherwise the $\Delta J$ values will be incorrect (see Step 11 below)
   - So, you need to know the cost per unit length for different pipe sizes
   - PVC pipe is sometimes priced by weight of the plastic material (weight per unit length depends on diameter and wall thickness)
   - You also need to know the annual interest rate upon which to base the calculations; this value will take into account the time value of money, whereby you can make a fair comparison of the cost of a loan versus the cost of financing it “up front” yourself
   - In any case, we want an equivalent uniform annual cost of the pipe over the life of the pipeline
   - Convert fixed costs to equivalent uniform annual costs, UAC, by using the “capital recovery factor”, CRF

   \[
   UAC = P(CRF) \tag{73}
   \]
\[ CRF = \frac{i(1+i)^n}{(1+i)^n - 1} \]  

(74)

where \( P \) is the cost per unit length of pipe; \( i \) is the annual interest rate (fraction); and \( n \) is the number of years (useful life)

- Of course, \( i \) could also be the monthly interest rate with \( n \) in months, etc.

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & n-1 & n \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \downarrow \\
A & & & & & \\
\end{array}
\]

- Make a table of UAC values for different pipe sizes, per unit length of pipe
- The CRF value is the same for all pipe sizes, but \( P \) will change depending on the pipe size
- Now you have the equivalent annual cost for each of the different pipe sizes

3. Determine the number of operating (pumping) hours per year, \( O_t \):

\[ O_t = \frac{\text{(irrigated area})(\text{gross annual depth})}{\text{(system capacity)}} = \text{hrs / year} \]  

(75)

- Note that the maximum possible value of \( O_t \) is 8,760 hrs/year (for 365 days)
- Note also that the “gross depth” is annual, so if there is more than one growing season per calendar year, you need to include the sum of the gross depths for each season (or fraction thereof)

4. Determine the pumping plant efficiency:

- The total plant efficiency is the product of pump efficiency, \( E_{\text{pump}} \), and motor efficiency, \( E_{\text{motor}} \)

\[ E_p = E_{\text{pump}}E_{\text{motor}} \]  

(76)

- This is equal to the ratio of “water horsepower”, WHP, to “brake horsepower”, BHP (\( E_{\text{pump}} = \text{WHP/BHP} \))
- Think of BHP as the power going into the pump through a spinning shaft, and WHP is what you get out of the pump – since the pump is not 100% efficient in energy conversion, WHP < BHP
- WHP and BHP are archaic and confusing terms, but are still in wide use
- $E_{\text{motor}}$ will usually be 92% or higher (about 98% with newer motors and larger capacity motors)
- $E_{\text{pump}}$ depends on the pump design and on the operating point (Q vs. TDH)
- WHP is defined as:

$$\text{WHP} = \frac{QH}{102} \quad (77)$$

where Q is in lps; H is in m of head; and WHP is in kilowatts (kW)

- If you use m in the above equation, UAC must be in $/100$ m
- If you use ft in the above equation, UAC must be in $/100$ ft

- Note that for fluid flow, “power” can be expressed as $\rho g Q H = \gamma Q H$
- Observe that $1,000/g = 1,000/9.81 \approx 102$, for the above units (other conversion values cancel each other and only the 102 remains)
- The denominator changes from 102 to 3,960 for Q in gpm, H in ft, and WHP in HP

5. Determine the present annual energy cost:

$$E = \frac{O_f C_f}{E_p} \quad (78)$$

where $C_f$ is the cost of “fuel"

- For electricity, the value of $C_f$ is usually in dollars per kWh, and the value used in the above equation may need to be an “average” based on potentially complex billing schedules from the power company
- For example, in addition to the energy you actually consume in an electric motor, you may have to pay a monthly fee for the installed capacity to delivery a certain number of kW, plus an annual fee, plus different time-of-day rates, and others
- Fuels such as diesel can also be factored into these equations, but the power output per liter of fuel must be estimated, and this depends partly on the engine and on the maintenance of the engine
- The units of E are dollars per WHP per year, or dollars per kW per year; so it is a marginal cost that depends on the number of kW actually required
6. Determine the marginal equipment cost:

- Note that $C_f$ can include the “marginal” cost for the pump and power unit (usually an electric motor).
- In other words, if a larger pump & motor costs more than a smaller pump, then $C_f$ should reflect that, so the full cost of friction loss is considered.
- If you have higher friction loss, you may have to pay more for energy to pump, but you may also have to buy a larger pump and/or power unit (motor or engine).
- It sort of analogous to the Utah Power & Light monthly power charge, based solely on the capacity to deliver a certain amount of power.

\[ C_f \text{ ($/kWh)} = \text{energy cost + marginal cost for a larger pump & motor} \]

where “marginal” is the incremental unit cost of making a change in the size of a component.

- This is not really an “energy” cost per se, but it is something that can be taken into account when balancing the fixed costs of the pipe (it falls under the operating costs category, increasing for decreasing pipe costs).
- That is, maybe you can pay a little more for a larger pipe size and avoid the need to buy a bigger pump, power unit and other equipment.

- To calculate the marginal annual cost of a pump & motor:

\[ \text{MAC} = \frac{\text{CRF}(\$\text{big} - \$\text{small})}{O_t(\text{kW}_{\text{big}} - \text{kW}_{\text{small}})} \]

(79)

where MAC has the same units as $C_f$, and $\$\text{big} - \$\text{small}$ is the difference in pump+motor+equipment costs for two different capacities.

- The difference in fixed purchase price is annualized over the life of the system by multiplying by the CRF, as previously calculated.
- The difference in pump size is expressed as $\Delta$BHP, where $\Delta$BHP is the difference in brake horsepower, expressed as kW.
- To determine the appropriate pump size, base the smaller pump size on a low friction system (or low pressure system).
- For BHP in kW:

\[ \text{BHP} = \frac{Q_s H_{\text{pump}}}{102E_p} \]

(80)
Round the BHP up to the next larger available pump+motor+equipment size to determine the size of the larger pump.

Then, the larger pump size is computed as the next larger available pump size as compared to the smaller pump.

Then, compute the MAC as shown above.

The total pump cost should include the total present cost for the pump, motor, electrical switching equipment (if appropriate) and installation.

Note that this procedure to determine MAC is approximate because the marginal costs for a larger pump+motor+equipment will depend on the magnitude of the required power change.

Using $S_{big} - S_{small}$ to determine MAC only takes into account two (possibly adjacent) capacities; going beyond these will likely change the marginal rate.

However, at least we have a simple procedure to attempt to account for this potentially real cost.

7. Determine the equivalent annualized cost factor:

This factor takes inflation into account:

$$EAE = \left[ \frac{(1+e)^n - (1+i)^n}{e - i} \right] \left[ \frac{i}{(1+i)^n - 1} \right]$$

(81)

where $e$ is the annual inflation rate (fraction), $i$ is the annual interest rate (fraction), and $n$ is in years.

Notice that for $e = 0$, $EAE = $ unity (this makes sense).

Notice also that the above equation has a mathematical singularity for $e = i$ (but $i$ is usually greater than $e$).

8. Determine the equivalent annual energy cost:

$$E' = (EAE)(E)$$

(82)

This is an adjustment on $E$ for the expected inflation rate.

No one really knows how the inflation rate might change in the future.

How do you know when to change to a larger pipe size (based on a certain sectional flow rate)?

*Beginning with a smaller pipe size (e.g. selected based on maximum velocity limits), you would change to a larger pipe size along a section of pipeline if the...*
difference in cost for the next larger pipe size is less than the difference in energy (pumping) savings

- Recall that the velocity limit is usually taken to be about 5 fps, or 1.5 m/s

9. Determine the difference in WHP between adjacent pipe sizes by equating the annual plus annualized fixed costs for two adjacent pipe sizes:

\[
E'(HP_{s1}) + UAC_{s1} = E'(HP_{s2}) + UAC_{s2}
\]

or,

\[
\Delta WHP_{s1-s2} = \frac{(UAC_{s2} - UAC_{s1})}{E'}
\]  

- The subscript \( s_1 \) is for the smaller of the two pipe sizes
- The units of the numerator might be $/100 m per year; the units of the denominator might be $/kW per year
- This is the WHP (energy) savings needed to offset the annualized fixed cost difference for purchasing two adjacent pipe sizes; it is the economic balance point

10. Determine the difference in friction loss gradient between adjacent pipe sizes:

\[
\Delta J_{s1-s2} = 102 \left( \frac{\Delta WHP_{s1-s2}}{Q_s} \right)
\]

- This is the head loss difference needed to balance fixed and annual costs for the two adjacent pipe sizes
- The coefficient 102 is for \( Q_s \) in lps, and \( \Delta WHP \) in kW
- You can also put \( Q_s \) in gpm, and \( \Delta WHP \) in HP, then substitute 3,960 for 102, and you will get exactly the same value for \( \Delta J \)
- As before, \( \Delta J \) is a head loss gradient, in head per 100 units of length (m or ft, or any other unit)
- Thus, \( \Delta J \) is a dimensionless “percentage”: head, \( H \), can be in m, and when you define a unit length (e.g. 100 m), the \( H \) per unit meter becomes dimensionless
- This is why you can calculate \( \Delta J \) using any consistent units and you will get the same result

11. Calculate the flow rate corresponding to this head loss difference:

- Using the Hazen-Williams equation:
\[ \Delta J = J_{s1} - J_{s2} = 16.42(10)^6 \left( \frac{q}{C} \right)^{1.852} \left( D_{s1}^{-4.87} - D_{s2}^{-4.87} \right) \] (86)

where \( q \) is in lps, and \( D \) is the inside diameter of the pipe in cm

- Or, using the Darcy-Weisbach equation:
  \[ \Delta J = \frac{800fq^2}{g\pi^2} \left( D_{s1}^{-5} - D_{s2}^{-5} \right) \] (87)

- Solve for the flow rate, \( q \) (with \( q \) in lps; \( D \) in cm):
  \[ q = C \left[ \frac{\Delta J}{16.42(10)^6 \left( D_{s1}^{-4.87} - D_{s2}^{-4.87} \right)} \right]^{-0.54} \] (88)

- This is the flow rate for which either size (\( D_{s1} \) or \( D_{s2} \)) will be the most economical (it is the balancing point between the two adjacent pipe sizes)
- For a larger flow rate you would choose size \( D_{s2} \), and vice versa

12. Repeat steps 8 through 11 for all other adjacent pipe sizes.

13. You can optionally create a graph with a log-log scale with the system flow rate, \( Q_s \), on the ordinate and the section flow rate, \( q \), on the abscissa:
   - Plot a point at \( Q_s \) and \( q \) for each of the adjacent pipe sizes
   - Draw a straight diagonal line from lower left to upper right corner
   - Draw a straight line at a slope of -1.852 (or -2.0 for Darcy-Weisbach) through each of the points
   - The slope will be different if the log scale on the axes are not the same distance (e.g. if you do the plot on a spreadsheet, the ordinate and abscissa may be different lengths, even if the same number of log cycles).
   - In constructing the graph, you can get additional points by changing the system flow rate, but in doing so you should also increase the area, \( A \), so that \( Q_1 \) is approximately the same as before. It doesn't make sense to change the system flow rate arbitrarily.
   - Your graph should look similar to the one shown below

- Find the needed flow rate in a given section of the pipe, q, make an intersection with the maximum system capacity (Q_s, on the ordinate), then see which pipe size it is
- You can use the graph for different system capacities, assuming you are considering different total irrigated areas, or different crop and or climate values
- Otherwise, you can just skip step 13 and just do the calculations on a spreadsheet for the particular Q_s value that you are interested in
- The graph is perhaps didactic, but doesn't need to be constructed to apply this economic pipe selection method

III. Notes on the Use of this Method

1. If any of the economic factors (interest rate, inflation rate, useful life of the system) change, the lines on the graph will shift up or down, but the slope remains the same (equal to the inverse of the velocity exponent for the head loss equation: 1.852 for Hazen-Williams and 2.0 for Darcy-Weisbach).

2. Computer programs have been developed to use this and other economic pipe selection methods, without the need for constructing a graphical solution on log-log paper. You could write such a program yourself.

3. The economic pipe selection method presented above is *not necessarily valid* for:
   - looping pipe networks
   - very steep downhill slopes
   - non-“worst case” pipeline branches
4. For loops, the flow might go in one direction some of the time, and in the opposite direction at other times. For steep downhill slopes it is not necessary to balance annual operation costs with initial costs because there is essentially no cost associated with the development of pressure – there is no need for pumping. Non-“worst case” pipeline branches may not have the same pumping requirements (see below).

5. Note that the equivalent annual pipe cost considers the annual interest rate, but not inflation. This is because financing the purchase of the pipe would be done at the time of purchase, and we are assuming a fixed interest rate. The uncertainty in this type of financing is assumed by the lending agency.

6. This method is not normally used for designing pipe sizes in laterals. For one thing, it might recommend too many different sizes (inconvenient for operation of periodic-move systems). Another reason is that we usually use different criteria to design laterals (the “20%” rule on pressure variation).

7. Other factors could be included in the analysis. For example, there may be certain taxes or tax credits that enter into the decision making process. In general, the analysis procedure in determining pipe sizes can get as complicated as you want it to – but higher complexity is better justified for larger, more expensive irrigation systems.
IV. Other Pipe Sizing Methods

- Other methods used to size pipes include the following:

  1. **Unit head loss method**: the designer specifies a limit on the allowable head loss per unit length of pipe
  2. **Maximum velocity method**: the designer specifies a maximum average velocity of flow in the pipe (about 5 to 7 ft/s, or 1.5 to 2.0 m/s)
  3. **Percent head loss method**: the designer sets the maximum pressure variation in a section of the pipe, similar to the 20%Pₐ rule for lateral pipe sizing

- It is often a good idea to apply more than one pipe selection method and compare the results
- For example, don’t accept a recommendation from the economic selection method if it will give you a flow velocity of more than about 10 ft/s (3 m/s), otherwise you may have water hammer problems during operation
- However, it is usually advisable to at least apply the economic selection method unless the energy costs are very low
- In many cases, the same pipe sizes will be selected, even when applying different methods

- For a given average velocity, \( V \), in a circular pipe, and discharge, \( Q \), the required inside pipe diameter is:

\[
D = \sqrt{\frac{4Q}{\pi V}}
\]

\( (89) \)

- The following tables show maximum flow rates for specified average velocity limits and different pipe inside diameters
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<thead>
<tr>
<th>Gallons per Minute</th>
<th>Litres per Second</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D (inch)</strong></td>
<td><strong>A (ft²)</strong></td>
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<tr>
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